

Prof. A. R. Rao Geometry Competition**Std. 10**

Date. 23-08-09

Time : 10-00 a.m. to 12-30 p.m.

- * There are two sections in the question paper. In the first hour questions of Section-I (First two pages) are to be attempted.
- * Section I is to be completed in first sixty minutes (1 hour). Write your answer in the bracket given against each question.
- * If you complete the first section early, then return the question paper back to the supervisor and ask for the questions of Section-II.

Name of the Student :

Seat No.:

Name of the School :

Section - I

- * There are two questions in the first Section.
- * Write answers in the question paper only and submit the question paper to the supervisor.

Q1 A In the following statements, write 'T' if the statement is true and 'F' if the statement is false, in the bracket given against each question. (16 Marks)

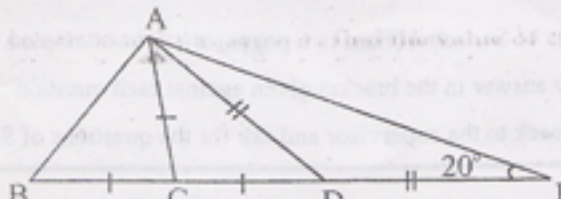
1. (a) If the radius of a circle is doubled, its perimeter is doubled. []
- (b) If the radius of a circle is doubled, its area is doubled. []
2. (a) If the length of a line segment is doubled, the number of points on the segment is also doubled. []
- (b) If the radius of a circle is doubled, the number of points inside the circle is doubled. []
3. (a) Two intersecting lines in space are coplanar. []
- (b) In space, two non-intersecting lines are parallel. []
4. (a) Diagonals of a square are congruent and perpendicular to each other. []
- (b) If the diagonals of a quadrilateral are congruent and perpendicular to each other, the quadrilateral is a square. []
5. (a) If the circumcentre of a triangle is in the exterior of the triangle, so is the orthocentre of the triangle. []
- (b) The circumcentre of a right angled triangle lies on one of its sides. []
6. (a) Sum of the measures of four interior angles of a quadrilateral is 360° . []
- (b) Sum of the measures of three exterior angles, one at each vertex, of a triangle is 360° . []
7. (a) Two triangles having a common base and lying between two parallel lines have equal area. []
- (b) If two triangles have a common base and have equal area, then they are lying between two parallel lines. []
8. (a) If two angles form a linear pair, then one of these angles is always obtuse. []
- (b) If an angle is not acute, it is obtuse. []

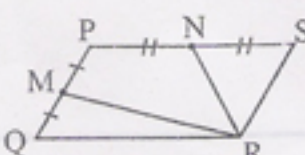
Q.1B Write your answer in the bracket given on the right side of the question. (12 Marks)

(Use the answer sheets if some rough work is necessary. Write only the answer in the question paper.)

1. Find the area of a regular hexagon inscribed in a circle of radius 1 unit. []

2. Given $4\sin\theta + 3\cos\theta = 5$, find the value of $\tan\theta$. []

3.  In the adjoining figure points B, C, D, E are collinear. $AC = BC = CD$ and $AD = DE$. If $m\angle AED = 20^\circ$ find $m\angle BAC$. []

4. In a $\square^m PQRS$, M and N are mid-points of \overline{PQ} and \overline{PS} respectively. If area of the $\square^m PQRS$ is 80, find the area of $\square PMRN$.  []

Q 2. Answer the following questions, selecting the proper choice from those given below each question.

Write your choice in the bracket given at the right side of each question. (10 Marks)

1. What is the measure of the angle between the minute hand and the hour hand of a clock at 10 minutes past 6 o' clock?

Choices : 110° , 115° , 120° , 125° Answer []

2. Four angles have the measures as given below, which of them can be constructed without use of a protractor.

Choices : 20° , 40° , 135° , 145° Answer []

3. In right angled triangle ABC, what is the value of $\sin^2 A + \sin^2 B + \sin^2 C$?

Choices : 1, $\sqrt{2}$, 2, 3 Answer []

4. How many small spherical balls of radius 1 mm can be prepared from a spherical ball of radius 1 cm. ?

Choices : 10, 100, 1000, 10000 Answer []

5. In a $\triangle ABC$, $AB : AC = 2 : 3$. Bisector of $\angle A$ intersects \overline{BC} in D. If $BC = 4$, then find DC.

Choices : 2.2, 2.4, 2.8, 3. Answer []

6. If $\sec\theta + \tan\theta = p$, then find the value of $\sec\theta - \tan\theta$.

Choices : $\frac{1}{p}$, $\frac{p}{p+1}$, $\sqrt{p^2-1}$, $\sqrt{1-p^2}$ Answer []

7. In a $\triangle ABC$, $m\angle A = 40^\circ$ and $m\angle B - m\angle C = 20^\circ$. Find $m\angle B$.

Choices : 40° , 60° , 80° , 100° Answer []

8. In a $\triangle ABC$, if $m\angle A = 2m\angle B = 6m\angle C$, then find $m\angle A$.

Choices : 100° , 108° , 110° , 112° Answer []

9. No angle of a $\triangle PQR$ has its measure less than 60° . Which type of a triangle is $\triangle PQR$?

Choices : Scalene, Isosceles, Equilateral, right angled Answer []

10. No angle of $\triangle ABC$ has its measure greater than 60° . Which type of a triangle is $\triangle ABC$?

Choices : Scalene, Isosceles, Equilateral, right angled Answer []

Section - II

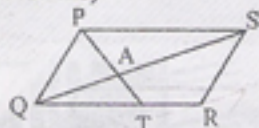
* There are four questions in this section. (Questions 3, 4, 5, 6)

* Answers are to be written in separate answersheets given to you.

Q. 3. Answer the following questions showing calculations briefly. (20 Marks)

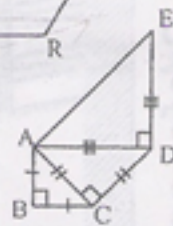
(1) In a $\square PQRS$, $T \in \overline{QR}$ is such that $QT : TR = 3 : 2$.

Suppose \overline{PT} and \overline{QS} intersect in A. Find $PA : AT$.



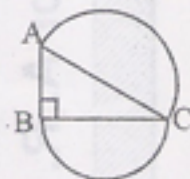
(2) In the figure $AB = BC = 2$. $m\angle B = m\angle ACD = m\angle ADE = 90^\circ$.

$AC = CD$ and $AD = DE$. Find area of the pentagon ABCDE.



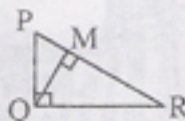
(3) In a $\triangle ABC$, $m\angle B = 90^\circ$. Areas of semicircles drawn on \overline{AC} and \overline{BC} as diameters are respectively 277 and 200 units. Find AB. ($\pi = \frac{22}{7}$)

(Hint : First, find the area of the semicircle on \overline{AB} as a diameter, then find AB.)



(4) In a $\triangle PQR$, $m\angle PQR = 90^\circ$. $\overline{QM} \perp \overline{PR}$, $M \in \overline{PR}$,

If $PQ = 2$ and $QR = 4$, find QM.



Q 4. Carry out the following constructions showing all steps of constructions with explanation.

Proofs of constructions are more important than accurate figures. (10 Marks)

(If you do not have the compass box, draw rough figures)

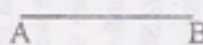
(1) An arc of a circle is given in the figure. Find the centre of the circle.

(Hint : Perpendicular bisector of any chord of a circle passes through the centre of the circle.)



(2) Assuming the length of a given segment \overline{AB} to be one unit, construct a segment of length $\sqrt{3}$ unit.

(Hint : Use Pythagoras theorem once or repeatedly.)



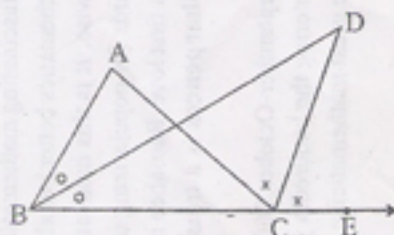
Q 5. Complete proofs of the following riders using the given hints.

It is not necessary to write 'Data' and 'To prove'. (12 Marks)

(1) In a $\triangle ABC$, the bisector of $\angle B$ and the bisector of $\angle ACE$,

as shown in the figure, meet at D. Prove that, $m\angle D = \frac{1}{2} m\angle A$.

Hint : Prove (i) $m\angle A = m\angle ACE - m\angle ABC$ (ii) $m\angle D = m\angle DCE - m\angle DBC$

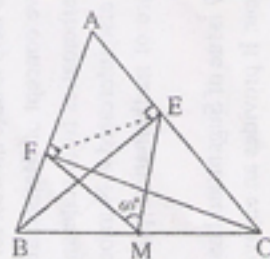


(2) \overline{BE} and \overline{CF} are the altitudes of a $\triangle ABC$. M is the mid-point of \overline{BC} .

If $m\angle EMF = 60^\circ$, prove that $EF = \frac{1}{2} BC$.

Hint : Show (i) $EM = \frac{1}{2} BC$ (Why?) Similarly $FM = ?$.

(ii) $\triangle EMF$ is an equilateral triangle. (Why?)

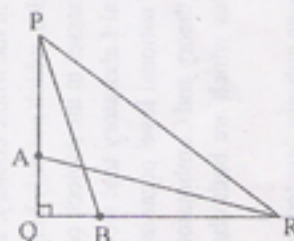


Q 6. Give proofs of the following riders. (20 Marks)

(1) In a $\triangle PQR$, $m\angle Q = 90^\circ$ and $A \in \overline{PQ}$, $B \in \overline{QR}$ are such that

$PA : AQ = RB : BQ = 3 : 1$.

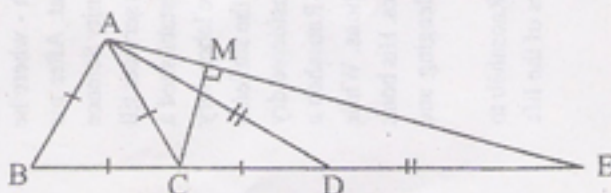
Prove that, $PB^2 + AR^2 = \frac{17}{16} PR^2$



(2) In the figure, B, C, D, E are collinear points.

$AB = BC = AC = CD$ and $AD = DE$. M is the foot of

the perpendicular from C on \overline{AE} . Prove that $AM = CM$.



પ્રાધ્યાપક એ. આર. રાવ



Professor A. R. Rao

રાવ સાહેબના હુલામણા નામથી જાણીતા પ્રો. એ. આર. રાવ 23મી સપ્ટેમ્બર 2009ના રોજ તેમના 100મી જન્મજયની એક સો એક વર્ષ પુરાં કરશે. ભારતના મુર્ધન્ય ગણિતજ્ઞ એવા રાવસાહેબ પા સદીથી વધુ સમય માટે જુનાગઢની બહાઉદ્દીન કોલેજમાં અને ત્યારબાદ ગુજરાતની અન્ય કોલેજોમાં અધ્યાપક તરીકે કાર્યરત રહ્યા. અધ્યાપન ક્ષેત્રથી નિવૃત્ત થયા બાદ રાવ સાહેબ અમદાવાદના વિક્રમ એ. સારાભાઈ કોમ્યુનિટી સાયન્સ સેન્ટરમાં ગણિત તજજ્ઞ તરીકે જોડાયા. આજ દિન પર્વત રાવસાહેબની સેવાઓ આ સેન્ટરને મળતી રહી છે. દેશભરમાં અજોડ એવી ગણિતની પ્રયોગશાળા રાવસાહેબે અહીં વિકસાવી છે. વિદ્યાર્થીઓનો ગણિતમાં રસ જાગૃત કરે અને જનસમાજની ગણિતમાં રુચિ ઉત્પન્ન કરે એવા અનેક ગણિતિક નમૂનાઓ (models), કોષ્ટકો અને રમતો રાવસાહેબે બનાવેલ છે. ગણિતને લોકભોગ્ય બનાવવા માટે રાવસાહેબ રાજ્ય અને રાષ્ટ્ર કક્ષાની અનેક સંસ્થાઓ દ્વારા સન્માનિત થયા છે. ગણિતમાં ભુમિતિ અને સંખ્યાશાસ્ત્ર રાવસાહેબના પ્રિય વિષયો છે. કોઈને પણ જડકી રાખે તેવા ગણિતના કોષ્ટકોનો અજાના સર્વ રાવસાહેબનું *Brain Sharpeners* નામનું પુસ્તક સૌ કોઈએ જોવા-માણવા જેવું છે.

સપ્ટેમ્બર 2007માં પ્રકાશિત સુગણિતનું પ્રો. એ. આર. રાવ વિશેષાંક તમે જરૂર વાંચો. ગણિત વિશે અને રાવસાહેબ વિશે તમને તેમાંથી ઘણું જાણવાનું મળશે.

Popularly known as Raosaheb, Prof. A. R. Rao will be completing a **hundred and one** years of his active and fruitful life on September 23, 2009. One of the leading mathematicians of the country, Raosaheb worked as a Professor of Mathematics, first in Bahauddin College, Junagadh - where he spent more than a quarter of a century - and then in various other colleges of Gujarat. After his retirement from active teaching in 1976, Raosaheb joined the Vikram A. Sarabhai Community Science Centre (VASCSC) at Ahmedabad, as a Professor Emeritus, where he continues to offer his services till today. Considered to be the father of nonformal mathematics in Gujarat, Raosaheb has established a Mathematics Laboratory at VASCSC, which is unique of its kind in the whole country. The laboratory is replete with fascinating mathematical models, games and puzzles, which not only help the students understand mathematics better but also help kindle interest for masses in the subject, considered dry and dull otherwise. It is this innovative concept of Mathematical Laboratory that won Raosaheb a number of awards for popularizing mathematics from state and national level organizations. While Geometry is his first love, Raosaheb is also interested in Number theory and Combinatorics. His book titled *Brain Sharpeners* is a treasure of mathematical puzzles which are both challenging and captivating.

The September-October, 2007 issue of Suganitam has been entirely devoted to Raosaheb to celebrate his entering the hundredth year. It provides an enjoyable reading with glimpses of the life and work of the great mathematician.